Twisted polynomials for knots and 3-manifolds with applications to concordance, slicing and fibering

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Abstract

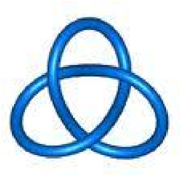
- The aim of my talk is to present the twisted Alexander polynomials and their applications to the topology of 3-manifolds.
- In the first part, after a short review of the Alexander and Jones invariants, I will present the twisted polynomials for knots and 3-manifolds after [Kirk-Livingston, Top. '99], [Wada, Top. '94] and [Lin, Columbia preprint '90].
- The second part is devoted to applications of the twisted polynomials to knots slicing, concordance and fibering mainly after [Kirk-Livingston, Top. '99] .
- In the third part, I'll present after [Friedl-Vidussi, Ann. of Math. '11] a criterion of fibering for 3-manifolds and also a nice corollary which relates fibered 3-manifolds with symplectic 4-manifolds.

Knots and links

- A knot K is an embedding of $\mathbb{S}^1$ in $\mathbb{S}^3$. A link L is a finite union of disjoint knots.
- There are two types of equivalence relations on links:
  1)weak equivalence $\sim$: $L_1 \sim L_2$ if there exists a homeomorphism of $\mathbb{S}^3$ which takes one in the other.
  2)equivalence $\approx$: $L_1 \approx L_2$ if they are ambient isotopic or equivalently there exists a homeomorphism of $\mathbb{S}^3$, which preserves the orientation and takes L_1 into L_2.
- In fact, two weakly equivalent knots are either equivalent or one of them equivalent with the mirror of the other.

Knots and links

For example the two trefoils are weak-equivalent, one is the mirror of the other, but not equivalent:



The aim of knot theory is to classify knots up to (weakly) equivalence. As usual in algebraic topology, one try to introduce invariants which can distinguish different classes of knots. For example a result of Gordon and Luecke states that the homeo type of the complement is a complete invariant for knots with respect to weak-equivalence (for links this is false).

Knots and links

- As far as it is difficult to decide when two knot complements are homeomorphic one associate weaker, but easier to compute invariants along the following recipe:

$$\textit{Homeo type of } (\mathbb{S}^3 \backslash K) \to \pi_1(\mathbb{S}^3 \backslash K) \to A_K \to \Delta_K,$$

  where A_K is the Alexander module and Δ_K is the Alexander polynomial.
- For a knot, $\Delta_K \in \mathbb{Z}[t, t^{-1}]$ is a polynomial in the Laurent ring, defined up to a multiplicative unit. In fact, it depends ONLY on π/π'' so it CANNOT detect chirality (i.e. a knot from its mirror).

The Jones polynomial

- A powerful invariant discovered in the $'90s$ is the Jones polynomial V_L defined for oriented links, introduced using the theory of operator algebras.
- It is also a Laurent polynomial but has the great advantage that sometimes it CAN detect chirality.
- For example, the two trefoils have the same Alexander polynomial, but are distinguished by the Jones's:

$$\Delta_{right-trefoil}(t) = \Delta_{left-trefoil}(t) = t^2 - t + 1$$

$$V_{right-trefoil}(t) = t + t^3 - t^4$$

$$V_{left-trefoil}(t) = t^{-1} + t^{-3} - t^{-4}.$$

Homology with local coefficients

- Let X a space which admits an universal cover $\tilde{X}$ and $\pi := \pi_1(X)$.
- Let A an abelian group and $\rho : \pi \to Aut(A)$ a representation which endow A with a $\mathbb{Z}[\pi]$-module structure.
- We shall define the homology of X with twisted coefficients in A.
- Let $C_*(\tilde{X})$-the simplicial or cellular chain complex of $\tilde{X}$. Since π acts on $\tilde{X}$ by covering transformations, each C_i is a $\mathbb{Z}[\pi]$ module. Denote $C_*(X, A) = C_*(\tilde{X}) \otimes_{\mathbb{Z}[\pi]} A$.
- Let $H_*(X, A_\rho)$-its homology. It is called the homology with local coefficients in A.

Examples

- For A-the trivial π-module we have: $H_*(X, A_\rho) = H_*(X, A)$.
- For $A = \mathbb{Z}[\pi]$ we have: $H_*(X, \mathbb{Z}[\pi]_\rho) = H_*(\tilde{X}; \mathbb{Z})$.
- For M an abelian group with trivial π-action and $A = \mathbb{Z}[\pi] \otimes_{\mathbb{Z}} M$ we have: $H_*(X, A_\rho) = H_*(\tilde{X}, M)$.
- (Shapiro's Lemma) If $H \triangleleft \pi$ is a normal subgroup and $A = \mathbb{Z}[\pi/H]$ then: $H_*(X, A_\rho) = H_*(X^H, \mathbb{Z})$, where X^H is the covering which corresponds to H.
- As a conclusion of the above examples, the twisted homology encodes various homologies of ALL coverings of X.

Twisted Alexander modules and polynomials

- Twisted Alexander polynomials were introduced, as in the classical case, by 3 approaches:
  -using Seifert surfaces by Lin in 1990
  -using Fox calculus by Wada in 1994
  -using homology with twisted coefficients by Kirk-Livingston in 1999.
- The last method starts with the following input data:
  - a surjection $\epsilon : \pi \to \mathbb{Z}$ with the associated cyclic covering X_∞
  - a field F and an F-vector space V
  - a representation $\rho : \pi \to GL_n(V)$ and the induced $\rho' : \pi' \to GL_n(V)$.

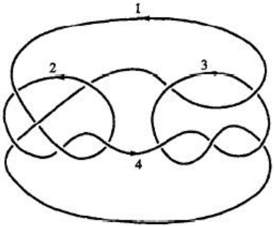
Twisted Alexander modules and polynomials

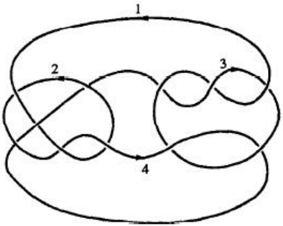
- The twisted homology construction is applied to X_∞ and ρ', producing homology groups $H_*(X_\infty, V_{\rho'})$ with $F[\mathbb{Z}]$-module structure. They are called the twisted Alexander modules.
- As $F[\mathbb{Z}]$ is a PID, one can consider the order of the torsion part of $H_i(X_\infty, V_{\rho'})$ denoted by $\Delta_i := \Delta_i(X, \epsilon, \rho, V) \in F[t, t^{-1}]$ called the i^{th} twisted Alexander polynomial.
- The $\Delta_i's$ are related with Wada's twisted polynomial W by the following formula:

$$\Delta_1 = W \cdot \Delta_0.$$

The Kinoshita-Terasaka and Conway knots

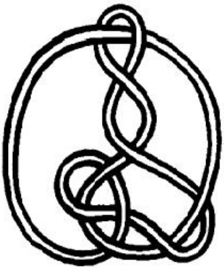
In the following, we have from Wada, an example of two knots, Kinoshita-Terasaka and Conway, with the same Alexander, Jones and HOMFLY polynomials but detected by the set of all twisted Alexander polynomials which corresponds to parabolic (i.e.the meridian goes to a trace 2 matrix) representations $\rho : \pi \to SL_2(\mathbb{F}_7)$:



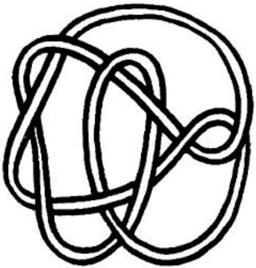


The 10_{40} and 10_{103} knots

Moreover, from [Friedl-Vidussi, Bull. Lond. Math. Soc. '07], the following two knots are detected by twisted polynomials even if they have, beside the same Alexander, Jones and HOMFLY, also the same Khovanov and knot Floer homology:



10_{40}



10_{103}

Applications to slicing

- A knot $K \in \mathbb{S}^3$ is slice if it bounds a smooth 2-disk in the 4-ball $\mathbb{B}^4$. An interesting problem is the construction of invariants that detects the sliceness of a knot. For example, a classical result, due to Fox and Milnor, asserts that for a slice knot, the Alexander polynomial has the form $f(t)f(t^{-1})$. The trefoil is therefore NOT slice.
- In [Kirk-Livingston, Top. '99] the twisted Alexander polynomials are used to produce a powerful obstruction to sliceness. Let's consider the following data:
  - an oriented knot $K \in \mathbb{S}^3$
  - for $m = p^r$ a prime power, B_m is the m-cyclic branched covering of $\mathbb{S}^3$ along K and E_m the m-cyclic covering of the complement $\mathbb{S}^3 \setminus K$
  - $\epsilon : H_1(E_m, \mathbb{Z}) \to H_1(\mathbb{S}^3 \setminus K, \mathbb{Z}) = \mathbb{Z}$ (here is hidden the orientation of K)

Applications to slicing

- for $d = q^s$ a prime power, a character $\chi : H_1(B_m, \mathbb{Z}) \to \mathbb{Z}_d$
As $\mathbb{Z}_d$ acts on $\mathbb{Q}(\xi_d)$ by $\xi_d := e^{\frac{2\pi i}{d}}$-multiplication, the last input is the representation
- $\rho_\chi : \pi_1(E_m) \to H_1(E_m, \mathbb{Z}) \to H_1(B_m, \mathbb{Z}) \to \mathbb{Z}_d \to GL_1(\mathbb{Q}(\xi_d))$.
The main obstruction to sliceness from [Kirk-Livingston, Top. '99] is:

Theorem

If K is a slice oriented knot and $m = p^r$, $d = q^s$ are odd prime powers, then there is a sub-group $M \subset H_1(B_m, \mathbb{Z})$ such that:
1) $| M |^2 = | H_1(B_m, \mathbb{Z}) |$
2) for all $\chi : H_1(B_m, \mathbb{Z}) \to \mathbb{Z}_d$ vanishing on M, the twisted polynomial $\Delta_1(E_m, \epsilon, \rho_\chi, \mathbb{Q}(\xi_d)) \in \mathbb{Q}(\xi_d)[t, t^{-1}]$ has the form

$$\begin{cases} at^n f(t)\overline{f}(t^{-1}) & \text{if } \chi \text{ is trivial} \\ at^n f(t)\overline{f}(t^{-1})(t-1) & \text{if } \chi \text{ is nontrivial} \end{cases}$$

Applications to slicing

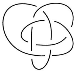
- A first consequence cf. [Herald, C., Kirk, P., Livingston, C, Math. Z. '10] of the above theorem concern the prime knots with at most 12 crossings.
- Among them only 175 were not ruled out by known invariants, and for several years the sliceness of 18 of these remain unknown.
- The above theorem settle 16 of these. From the last two, the 12_{a990} is proved to be slice, while the 12_{a631} is still unknown.
- Other applications are obtained in [Kirk, Livingston, Top. '99] using the relation between sliceness and concordance. Denote by m the mirroring operation, the $*$ means changing the knot orientation and $\sharp$ is the connected sum between oriented knots.

Applications to concordance

Definition

Two oriented knots K_1 and K_2 are concordant if one of the following (equivalent) conditions is verified:
1) there is an embedded cobordism $\mathbb{S}^1 \times [0,1] \in \mathbb{S}^3 \times [0,1]$ between K_1 and $m(K_2)^*$.
2) $K_1 \sharp m(K_2)^*$ is slice.

Kirk-Livingston used twisted polynomials in convenient representations to prove that the 8_{17} knot below, is chiral and not invertible.



Moreover, the above theorem applied to $8_{17} \sharp m(8_{17})$ show that 8_{17} is not even concordant with its inverse.

Applications to knot fibering

Definition

A knot $K \subset \mathbb{S}^3$ is fibered if its complement is a fibration over $\mathbb{S}^1$.

For example, the trefoil and figure eight knots are fibered. A classical result is the following:

Theorem

For a fibered knot K, the Alexander polynomial $\Delta_K(t)$ is monic of degree $2g(K)$.

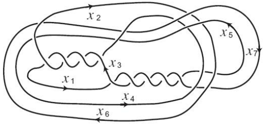
The following theorem from
[Kitano-Morifuji, Ann. Scuola Norm. Sup. Pisa '05] is a twisted version:

Applications to knot fibering

Theorem

For a knot K, a field F and a nonabelian representation $\pi_1(\mathbb{S}^3 \setminus K) \to SL(2, F)$, the Wada twisted polynomial (which by definition is a rational function) is a polynomial. Moreover, if K is fibered of genus g, then $W(t)$ is monic of degree $4g - 2$.

For example, the knot below, has a nonabelian $SL(2, \mathbb{F}_7)$ representation. The Alexander and Wada polynomials are: $\Delta(t) = t^4 - t^3 + t^2 - t + 1$ $W(t) = t^4 + 6t^3 + 6t^2 + 6t + 1$. If it were fibered the genus would be 2 from Δ and 3 from W. So it is not fibered.



Applications to hyperbolic torsion

- In the hyperbolic setting, there are at least two directions where the ideas of twisted polynomials were applied.
- In [Dubois-Yamaguchi, '09], for hyperbolic 3-manifolds, the authors gives a derivative formula for the hyperbolic Reidemeister torsion, in terms of the twisted Alexander polynomials.
- For hyperbolic knots, (i.e. with hyperbolic complement) in [Dunfield-Friedl-Jackson, '11], using the twisted polynomials, a new invariant T_K- the hyperbolic torsion polynomial - is defined. In particular, the following theorem is proved:

Theorem

$T_{m(K)} = \bar{T}_K$ for any hyperbolic knot K, and so if K is amphichiral, $T_K(t)$ is a real Laurent polynomial.

Applications to hyperbolic torsion

- Another interesting result from the last paper is concerned with knots with at most 15 crossings.
- There are 313.231 such prime knots and except 22, all are hyperbolic.
- For the hyperbolic ones, $T_K(t)$ is a powerful invariant, fully detecting the chirality.
- Morover, in many cases, $T_K(t)$ gives sharp informations on the genus and fibering.

Fibered 3-manifolds

The main result of [Friedl-Vidussi, Ann. of Math. '11] is a description of fibered 3-manifolds in terms of twisted polynomials.

- A manifold pair (N, φ) is composed by a connected orientable 3-manifold N with toroidal or empty boundary and a nontrivial morphism $\varphi : \pi_1(N) \to \mathbb{Z}$.

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- A manifold pair FIBERS over $\mathbb{S}^1$ if there is a fibration $p : N \to \mathbb{S}^1$ with the induced map $\pi_1(N) \to \pi_1(\mathbb{S}^1) = \mathbb{Z}$ being φ.

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- For a manifold pair, the Thurston norm is $\parallel \varphi \parallel = min\{\chi_-(S) \mid S \textit{ surface Poincare dual to } \varphi\}$, where $\chi_-(S) = \Sigma max\{-\chi(S_i), 0\}$ for $S = S_1 \cup ... \cup S_k$.

Fibered 3-manifolds

- For $\alpha : \pi_1(N) \to G$ a morphism ONTO a finite group, and the induced $\pi_1(N) \to G \to Aut(\mathbb{Z}[G])$ we denote:
  - $\Delta^{\alpha}_{N,\varphi} \in \mathbb{Z}[t, t^{-1}]$ the twisted Alexander polynomial,
  - φ_α the restriction of φ to the kernel of α,
  - $div\ \varphi_\alpha = max\{n \in \mathbb{N} \mid \varphi_\alpha = n \cdot \psi\}$.

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- With the above notations the Friedl-Vidussi main result is:

Theorem (Fundamental theorem)

For a manifold pair (N, φ) the following are equivalent:
1. (N, φ) is fibered
2. For any morphism $\alpha : \pi_1(N) \to G$ onto a finite group G, $\Delta^{\alpha}_{N,\varphi}$ is monic of degree $\mid G \mid \cdot \parallel \varphi \parallel + (1 + b_3(N)) \cdot div\ \varphi_\alpha$.

Fibered 3-manifolds and symplectic 4-manifolds

From the above theorem, they prove also the:

- **Theorem**

  For N a closed 3 manifold, the following are equivalent:
  1. $\mathbb{S}^1 \times N$ is symplectic
  2. N is fibered.

Fibered 3-manifolds and symplectic 4-manifolds

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- **Theorem**

  For N a closed 3 manifold, the following are equivalent:
  1. $\mathbb{S}^1 \times N$ *is symplectic*
  2. N is fibered.

  - For this second theorem, the $2 \Rightarrow 1$ part, was known by Thurston. The $1 \Rightarrow 2$ direction consist in the proof that using the symplectic structure, one can construct a class $\varphi \in H^1(N, \mathbb{Z}) = Hom(\pi_1(N), \mathbb{Z})$ which satisfy the conditions in the Fundamental theorem. Along the way, deep results of Taubes, Meng and Donaldson are used.

The fundamental theorem

- The fundamental theorem appeared first on arxiv in ′08 and after that in Annals in ′11.
  The $1 \Rightarrow 2$ part, was proved in various levels of generality by Cha, Goda, Kitano, Morifuji and finally Friedl-Vidussi. The main tools are a Mayer-Vietoris sequence and the calculus of the twisted polynomials by a Seifert-type method.
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  For the $2 \Rightarrow 1$ part, the following main steps are described by Friedl-Vidussi in an expository paper from arxiv in ′10.
- **STEP A** is essentially the following theorem:

Theorem (A)

There is a connected surface S minimizing the Thurston norm , and the pair pair (N, φ) fibers over $\mathbb{S}^1$ iff $i_{\pm} : \pi_1(S) \to \pi_1(N \setminus S)$ are isomorphisms.

The fundamental theorem

- **STEP B** In view of the STEP A the idea is to interpret the information on the twisted polynomials in terms of the $i_{\pm}$. The main result here is:

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- Theorem (B)

  If $\alpha : \pi_1(N) \to G$ *is a morphism ONTO a finite group such that* $\Delta^{\alpha}_{N,\varphi} \neq 0$ *and it verify the hypothesis of the fundamental theorem, then* $i_{\pm} : H_1(S, \mathbb{Z}[G]) \to H_1(N \setminus S, \mathbb{Z}[G])$ *are isomorphisms.*

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- In the untwisted case, theorem B says that if the Alexander polynomial verifies the hypothesis from the fundamental theorem, then $i_\pm : H_1(S, \mathbb{Z}) \to H_1(N \setminus S, \mathbb{Z})$ are isomorphisms, or in other words that $i_\pm : \pi_1(S, \mathbb{Z}) \to \pi_1(N \setminus S, \mathbb{Z})$ are isomorphisms at "abelian level".

The fundamental theorem

- **STEP C** In view of the above remark, one can say that this step is devoted to the "finite solvable level":

Theorem (C)

If for any $\alpha : \pi_1(N) \to G$ a morphism ONTO a finite solvable group, $\Delta^{\alpha}_{N,\varphi}$ verify the hypothesis of the fundamental theorem, then for any finite solvable group G, the induced maps $i_\pm : Hom(\pi_1(N \setminus S), G) \to Hom(\pi_1(S), G)$ are bijections.

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- To go further, we need the following:

Definition

If P is a property of groups (finite, solvable, etc), we say that a group π is *residually* P if for any nontrivial $x \in \pi$ there is a group G with P and a morphism $\alpha : \pi \to G$ such that $\alpha(x)$ is nontrivial.

The fundamental theorem

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Theorem (D)

If the fundamental theorem holds for all 3-manifolds N with $\pi_1(N)$ residually finite solvable, then it holds for all 3-manifolds.

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If the fundamental theorem holds for all 3-manifolds N with $\pi_1(N)$ residually finite solvable, then it holds for all 3-manifolds.

- **STEP E** This last step proves the following:

Theorem (E)

If $\pi_1(N \setminus S)$ is residually finite solvable, and for any finite solvable G, the maps $i_\pm : Hom(\pi_1(N \setminus S), G) \to Hom(\pi_1(S), G)$ are bijections, then $N \setminus S$ is a product.

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The fundamental theorem

- Putting all together, by theorem D one can suppose π_1 residually finite solvable.
- In this case, by theorem C, are verified the hypothesis of theorem E.
- Hence, by theorem E, S and $N \setminus S$ are homotopic equivalent.
- Therefore, by theorem A, N is fibred.

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