

A homological model for the coloured Jones polynomials

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Abstract

In 1991, Reshetikhin and Turaev defined a method that starts with a quantum group and leads to link invariants. This construction is purely algebraic and combinatorial. The coloured Jones polynomials $J_N(L, q)$ are a family of quantum invariants for links constructed using this method from the representation theory of the quantum group $U_q(sl(2))$. The first invariant from this sequence, corresponding to the parameter $N = 2$ is the original Jones polynomial $J(q)$. This is a quantum invariant, but it can be characterized also by skein relations. We will describe a homological model for all coloured Jones polynomials.

R. Lawrence defined a sequence of representations of the braid groups, using the homology of a certain covering of a configuration space. From these homological representations, Bigelow [2] and Lawrence [5] described the original Jones polynomial as a graded intersection pairing in a covering of a configuration space of the punctured disk, using the skein nature of the invariant for the proof.

In 2012, Kohno ([4], [3]) proved a deep connection between the representations of the braid group on the highest weight spaces of $U_q(sl(2))$ -modules and the Lawrence representations. We present a homological model for $J_N(L, q)$, using this identification and the quantum nature of the invariant for the proof. We show that any coloured Jones polynomial can be described as a graded intersection pairing between two homology classes on a covering of the configuration space of the punctured disc.

This work is available at [1].

References

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